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Revisiting Output Employment Relationship in Pakistan: An Empirical Investigation of Okun's Law Using Rolling Regression

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ABSTRACT

Economists have proven the negative relationship between GDP growth in the economy and the unemployment rate changes over time. This study uses the empirical application of Okun's Law using time varying coefficients to test the relationship between economic growth and rate of unemployment over time. The study used time series data for the period from 1991 to 2023 from Pakistan, which was sourced from World Bank. In empirical analysis, the augmented dickey fuller test is employed to assess the unit root of the variables. Based on unit root results, Autoregressive Distributed Lag (ARDL) bounds testing approach followed by Error Correction Model (ECM) utilized to investigate short term and long-term significance of Okun's law in Pakistan. The Okun's law with time varying effects are estimated using the rolling regression approach. The ECM results demonstrates that there is a significant and negative relationship between GDP growth and unemployment in Pakistan in the long run, but the time varying coefficients reveals that Okun's Law is not consistent over the analysis period. This work contributing to the corpus of knowledge in Pakistan by estimating Okun's law using rolling regression. According to the study's results, the Okun's law in Pakistan is not stable over the study period. There is a need to develop employment generating policies other than only focusing to economic growth. The policy intuitions can consider FDI, Trade Openness, and financial development as key indicators for reducing unemployment in Pakistan.

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1. Introduction

Achieving full employment is just one of the primary crucial macroeconomics goals for policy makers. According to ILO (2001) the definition of employed is "be doing work for wages, revenue, or household gain during the previous week, for at least one hour on any given day" and assist in working for household income at a family business or farm household throughout the previous week" as well as "have a job or enterprise such as a super market, industry, agriculture, or service formation, even if had not worked prior week for some purpose". According to ILO (2012) the definition of employed is "unemployment is a condition of being out of job or needing a work and continuously searching for it in the last four weeks or underemployed (age 16 or above) but can start working in the coming two weeks unemployed persons do not include people who do not desire to work, full-time students, retired persons, or children." Shahid (2014) analyzed that creating jobs is a keystone of any economic recovery program. Economically, employment benefits low-income people by generating income, supporting overall growth, and renewing domestic consumer demand for products and services. The gross domestic product is the overall actual amount of completed products and services generated within the borders of a nation for a specific year. According to the study by Oloni (2013), the author made a variety of claims and made arguments on the influence and connection between employment and economic growth that have backed up many employment-related claims. According to Hussain, Siddiqi and

Iqbal (2006) and Oloni (2013), increased economic growth in any nation produces a sizable increase in GDP and worker productivity, which can be helpful in enhancing job prospects in the nation. The same, Ibragimov and Ibragimov (2017) have also supported that the growing economy will help to the rapid growth of the workforce, lowering the country's unemployment rate.

According to Al-Habees and Rumman (2012) the government and policymakers have devised a variety of strategies that can help increase employment rates and raise people's standard of living and quality of life. Therefore, as result, it is possible that countries may develop stronger economic growth strategies that will benefit the public in terms of job possibilities and contributing to the country's overall success and profitability. It is widely known in economics that a country's GDP growth rate causes employment to rise and unemployment to fall. A theoretical idea known as "Okun's Law" connects output with unemployment. One of the most widely used correlations within Keynesian model has also been proven for a variety of nations and divisions, primarily in industrialized nations (Arif, 2020; Christopoulos, 2004; Lee, 2000). According to Ibragimov and Ibragimov (2017) analysis, economists and decision-makers have concentrated on the connection between unemployment and growth over the past 50 years. The economist Arthur Okun created a hypothesis based on factual evidence at the beginning of the 1960s, which is known as law. The macroeconomic health of the nation is mostly determined by the output and unemployment rates. When unemployment increases, fewer people are employed, which lowers both labor supply and output of products and services, as well as demand for commodities. The growth of the economy and the rate of decline in rates of unemployment are closely related, according to Arif (2020), who examined the correlation between the two variables. A rise in the growth rate causes employment to rise or unemployment to fall. Okun theory, which states that fluctuations in GDP and the rate of unemployment are inversely proportional, has been utilized in economic research to examine the relationship of economic growth and unemployment. According to the analysis, each 1% decrease in unemployment will yield either a 3% gain in real gross domestic product (RGDP), or inversely.

Knotek II (2007) examines the relationship between output and unemployment using a time varying coefficient (TYC). Both Knotek II (2007) and Wang, Li and Huang (2008) estimated the smooth time-varying coefficient using rolling regression as well as time-varying parameter for Okun by using the Bayesian Spline Approach. Additionally, the main idea that the Okun's relationship might structurally evolve was investigated (Lee, 2000; Sogner & Stiassny, 2002). Several researchers calculated the change in the Okun’s coefficients throughout time using rolling regression (Knotek II, 2007). Economists have shown over the past ten years that the inverse link between economic growth and unemployment rate is not constant. The key tool for quantifying such a relationship will be rolling regression. Specifically, for Zanin and Marra (2012) rolling regression is an investigation of how associations between variables change over time. Although time-varying coefficients are typically produced via rolling regression, the estimates within the given samples are constant. The Phillips curve also predicts an inverse connection among both inflation and unemployment, which has been confirmed by the findings of Zivanomoyo and Mukoka (2015). In the short run, the Phillips curve from 1958 demonstrates a negative link between inflation and unemployment. According to Al-Habees and Rumman (2012) private and public sectors both make fixed investments. Gross Domestic Fixed Capital Formation, or the total amount of money invested in new capital goods during a certain time period, is the most fundamental definition of fixed investment. Gross domestic fixed capital production is calculated by subtracting total investment from capital consumption. In a wide range of nations, Dutt, Mitra and Ranjan (2009) examined how trade policies impacted the overall unemployment rate. These studies find strong proof that open trade policies lower unemployment. Despite the high unemployment rate and sluggish economic development. Taraki and Arslan (2019) analyses that, net capital formation can be increase human capital and production while also presenting job opportunities. Growth is essential for both industrialized and rising nations. Long-term growth and measures to create jobs will finally end unemployment. Foreign direct investment is considered as beneficial to growing economies since it creates cross-border job opportunities (Hussain, Siddiqi, & Iqbal, 2006). According to Okun's Law, rising economic activity creates more employment options, which lowers overall unemployment. A country's exports rise because of increased economic growth, helping it to solve its balance of payments issue. Inflation is kept under control by increasing output and maintaining a stable exchange rate. Because of this, there is a lot of literature on predicting unemployment rates. For example, check Elliott and

Timmermann (2008); Rothman (1998), or, recent time, Franses, McAleer and Legerstee (2014). The focus of this study is to test a time varying effect using rolling regression.

1.1. Objectives of the study

a) To estimate relationship between unemployment and GDP growth using Okun's law in case of Pakistan.
b) To test a time varying effect using rolling regression.

The study's importance is determined by two factors: the very first aspect will be literature point of view. There are number of studies we are explored the Okun's law empirical studies such as Funke (1992); Wang, Li and Huang (2008). The second aspect will be the growth-oriented policies that we are applying for increasing employment are not directly applicable. Policy making for increasing employment level would be change not only depending on growth. In this study significant results will contribute to policy making. In this study we are going to test in case of Pakistan Okun's law run by testing time varying coefficient that will show as the difference applicability of Okun's law in case of Pakistan over the period. Further sections of the study presents the literature review, methodology, results and discussion, and conclusion.

2. Literature Review

Many studies have been done in the past to analyze the rate of unemployment and potential (GDP). Okun's study resulted in the development of a statistical link between fluctuations in output and variations in unemployment that can be used for policy analysis. Most people concluded that the Okun trades were unequal Adanu (2005); Ibragimov and Ibragimov (2017) examined the unemployment rate and output that was crucial indicators of the country 's macroeconomic health. When unemployment rises, the number of workers falls, lowering not available demand for commodities, but also in terms of the production of products and services. Okun's law, a fundamental theory defined the observed relationship between changes in the unemployment rates and rates of growth in real GDP. According to Shahid (2014), unemployment is defined as the number of unemployed persons in a given economy, frequently measured as a given proportion of the working population. Economic growth, as described by, is the process through which a nation's real annual income grows over time as judged by the quantity of goods and services consumed. According to Lal et al. (2010) in contrast to the other method of Okun's law, which focuses on the correlation among both rate of unemployment and GDP and claimed that 1% increases in unemployment leads in a 2% loss in GDP, the original version of Okun's law clearly indicated that when rate of unemployment falls by 1%, GDP growth rate increases by 3%. Kim, Yoon and Jei (2020) examined the parametric method. The results showed that Okun's legislation is not applicable in other nations, expect for Malaysia, Thailand, the Philippines. The time varying coefficient followed the same pattern as the detrend GDP. This revealed that because the economic growth of the ASIAN-6 countries was so largely dependent on foreign investment, the inverse correlation among the rate of unemployment and output intensified during a recession.

Furthermore, Kim, Park and Jei (2015) demonstrated that Okun's law is invalid in Nigeria, as like it is in many other countries that export natural resources in Africa. The study conclusions were supported by a cointegration analysis of information from 1980 to 2008. The same outcome was recently achieved by Arif (2020), demonstrating the significance of Okun's law as established by two prior studies. The study also noted that the link between rate of unemployment and inflation, FDI as well as the percentage of loans to the private sector is inverse. Okun's coefficient was largely studied in past analysis for the US and Europe. Okun's law was investigated in the United States by Attfield and Silverstone (1997), who validated Okun's general principle. Binet and Facchini (2013) in contrast side, established that Okun's law was applicable in Greece, Spain, and France. In addition, Lee (2000); Virén (2001) Real output and rate of unemployment are negatively correlated within OECD nations under study. According to Arif (2020) Okun's law gave a simple and efficient way to analyze and forecast variations in Australia's unemployment rate. In addition, Kim, Park and Jei (2015) looked at the Okun's law in the south Asian four countries (Singapore, Japan, Hong, Kong and Korea). Lal et al. (2010) examined the negative link among both economic growth and rate of unemployment in the five East Asia countries such as China, Pakistan, India, Bangladesh, and Sri Lanka. The study also claimed that several countries, like Pakistan, Bangladesh, Sri Lanka, and India are unable to address the issue of rising unemployment rates because of a lack of trustworthy foreign investment. However, the coefficients value is just enough to implement Okun's law. Louail and Benarous (2021) looked at

the economy of Algeria. For the years 1991 through 2019, annual economic data for Algeria were used. The gap version of Okun's coefficient was utilized in combination with the autoregressive distributed lag bounds testing technique model. The empirical findings suggest that Okun's law applies true in the Algerian economy.

Louail and Benarous (2021) examined Okun's law in the Saudi economy by investigating the influence of the output gap on unemployment rates as well as the role of economic growth to lowing unemployment. The goal of this research was to address a significant vacuum in the empirical studies on Okun's legislation drivers in the context of developing nations such as Arab nations specifically. The autoregressive distributed lag bounds testing approach was utilized to assess the existence of Okun's law in the Saudi economy between 1991 and 2017. The empirical findings show that Okun's law is applicable to the Saudi economy. Hussain, Siddiqi and Iqbal (2006) explored the link among rate of unemployment and the economic growth in Pakistan. As a time, series, the study used data between 1972-2006. Researchers used the ADF test, and because all the variables were stable at the first level of difference, the research employed Johansen Co-integration to determine the variables long-run associations. The long-run relation among both GDP growth, unemployment, labor, capital, and trade openness is revealed by the Co-integration test results. Overall, the results point to an inverse relationship between both GDP growth and unemployment. In case of Pakistan many literatures are available on Okun's law Arif (2020) demonstrated the link among the unemployment rate as well as the rate of production growth. To carry out this multivariate time series analysis, Pakistani data from 1972 to 2019 was utilized. To validate the integration order, the augmented dicky fuller test was utilized. This article shows in inverse relation between output growth and the unemployment rate in Pakistan. Javed determined the correlation between both unemployment and GDP growth in 2010. The study examined time series data on unemployment and GDP since 1981 to 2005 to evaluate the applicability of Okun's law in Pakistan. The study applied a modified technique of Okun's law. To establish the long-run connection, the study employed the Engle Granger co-integration approach, and the error correction method to determine GDP short run trend. The study observed that in the long term, GDP growth will adapt to the equilibrium more quickly.

Mahmood et al. (2014) collected data in Pakistan between 1983 to 2010. The econometric analysis used ordinary least squares. Inflation, GDP, FDI and rate of unemployment are some of the elements influencing this research. According to the results of a study, both; rate of economic growth and inflation have influence on employment. Pakistan's population and foreign direct investment have also been proven to have a substantial and negative impact on unemployment. Besides Hussain, Siddiqi and Iqbal (2006) investigated how Pakistan's unemployment rate and GDP growth interacted. The researcher used time series data spanning the years 1960 to 2012. The Johnson co-integration test was utilized after using the Augmented Dickey-Fuller test, which had stationary findings for the first difference. According to their research, a 1 percent increase rates of unemployment by 0.63 percentage points. A 1% reduction in unemployment, in contrast, results in a 7.25 percent rise in GDP growth. According to the research, long-term GDP growth is associated negatively with long-term unemployment. Besides, Akram et al. (2014) empirically evaluated Okun's law. To determine whether Okun's law applies to Pakistan's economy, the law's coefficient was calculated. Utilizing time series data from Pakistan's actual GDP and rate of unemployment, the Okun's law applicability was examined. The information was gathered from 1972 to 2012. The ordinary least squares approach was used in the analysis. Okun's law does not apply to the Pakistani economy, based on the actual findings. To summarize the preceding examination of the literature, it is evident that the key variables under discussion are simply the rate of unemployment and GDP growth, while several versions of Okun's coefficient are computed employing empirical data from various nations over different sample periods. The study underlines that Okun's law is not always applicable in Pakistan, and that unemployment is frequently linked to factors such as the economy's structure, labor market legislation, and the availability of skills. It's also worth noting that Okun's law seems to apply to signifies the fact and regularity, as well as decomposition and estimating methods.

3. Methodology

Okun's law has varied throughout time to reflect current employment trends and the state of the economy at the time. Zanin and Marra (2012) determined that, while one model of Okun's law concentrates on the correlation among both unemployment and GDP, indicating that because a 1% rise in unemployment resulted in a 2% drop in GDP, the other version concentrates on the

connection between unemployment and GNP, stating that a 1% decrease in unemployment results in a 3% increase in GNP. Let Y_t and Y_t^* showed the existing and prospective GNP logs, respectively. Let, U_t and U_t^* *demonstrates* the empirical and actual levels of unemployment. The "gap" equation of the following sort best describes the empirical link between unemployment and output that Okun proposes.

$$Y_t^c = \text{a}.U_t^c + \omega_t \text{ with } \alpha < 0 \quad (1)$$

Where $Y_t^c = Y_t - Y_t^*$ is the log of the annual GNP output value, $U_t^c = U_t - U_t^*$ is the seasonal unemployment rate, and differential between the observable and actual unemployment rates, and ω_t is a random error term. Okuns coefficient is the parameter used in equation (1). Knotek's basic difference interpretation of Okun's Law will be discussed.

$$\Delta U_{i,t} = \beta_0 + \beta_1 \text{GD}Pg_{i,t} + a_i + \varepsilon_{i,t} \quad (1.1)$$

The primary explanatory variable is GDPg (i,t). The estimate β_1 ought to have demonstrated the fundamental connection among GDP growth rate and changes in unemployment. This is predicted to be negative, implying that as the rate of growth accelerates, employment should accelerate. The model's unobserved time-invariant components are represented by the variable a_i. In this paper we intend to look through into legality of okuns law in Pakistan using rolling regression. We will use the Auto Distributed Lag Model, which is the closest method of Okun's law's Dynamic version method. Functional form of the model is:

$$U_t = \beta_0 + \beta_1 \text{GDP} + \beta_2 FDI + \beta_3 GFCF + \beta_4 IN + \beta_5 T + \varepsilon$$

Where,
U_t= Rate Of Unemployment In t Period
GDP= (Gross Domestic Product)
FDI= (Foreign Direct Investment)
GFCF= (Gross Fixed Capital Formation)
IN= Inflation and T= Trade % GDP

The following time varying model may be used to confidently estimate Okun's parameter.

$$\Delta y_t = \alpha + \beta_t(\Delta x_t / Yx_{t-1}) + \varepsilon_t, \qquad \beta_t = \text{s(t;}\delta\text{)}. \quad (1)$$

In which s(t;δ) is an unidentified smooth function with parameter vector δ. This approach has the benefit of being completely Smooth, with such a form determined by the data rather than the investigator's linear equation. According to Hastie and Tibshirani (1993). It should be noted that a time-varying coefficient theory is a subset of such a changed coefficient model with time as the effect modification. It's worth nothing that using a smooth expression in equation (1) is necessary because any functional form of relationship is relay recognized a prior; thus, imposing any structure (e.g., liner or quadratic) on it makes no sense; instead, we should let the data indicate whether the link is linear or nonlinear, and which counties are affected. The ARDL bound test was conducted after determining an appropriate lag to evaluate the presence of a long-run connection amongst the dependent variable and the independent variables. To avoid false results, the augmented dickey fuller test was performed to measure stationary. The ADF test results indicated that trade, FDI, and GFCF are I(1) and U_t, GDP and INF are I(0). As a result, the variables in the model are a combination of I (1) and I(0)variables, whereas there is no variable process of integration of order I(2). As more the study used the ARDL technique of cointegration, as proposed by Pesaran, Shin and Smith (2001), to analyze the short run versus long connection between the rate of unemployment and GDP. The ARDL technique's initial step is to apply an unconstrained VAR model and the Schwarz Information criteria and Akaike Information Criteria to figure out the best lag. In this study investigation, the SIC and AIC findings indicate that 3 lags are appropriate for both models to run the ARDL model. In the second phase, the study used the Bound test to see if weather variables used to have a long-run and short-run relationship between each other.

$$DU_t = \alpha_1 + \alpha_{2i} \sum_{i=1}^{3} DU_{t-i} + \alpha_{3i} \sum_{i=0}^{3} DGDP_{t-i} + \alpha_{4i} \sum_{i=0}^{3} DFDI_{t-i} + \alpha_{5i} \sum_{i=0}^{3} DGFCF_{t-i} + \alpha_{6i} \sum_{i=0}^{3} DTRADE_{t-i} + \alpha_{7i} \sum_{i=0}^{3} DIN_{t-i} + \alpha_{8i} \sum_{i=1}^{3} U_{t-i} + \alpha_{9i} \sum_{i=1}^{3} GDP_{t-i} + \alpha_{10i} \sum_{i=1}^{3} FDI_{t-i} + \alpha_{11i} \sum_{i=1}^{3} GFCF_{t-i} + \alpha_{12i} \sum_{i=1}^{3} TRADE_{t-i} + \alpha_{13i} \sum_{i=1}^{3} IN_{t-i} \varepsilon_t \quad (2)$$

D is the first regression coefficient in the preceding model, and GDP, FDI, GFCF, IN and TARDE are the five macroeconomic variables selected in the study. In this model, U_t is the dependent variable , with GDP, FDI, GFCF, IN, and TRADE as the long run regressors. The cointegration of the variables is determined through an ARDL bounds test. It implies comparing the F-statistic to critical boundaries, i.e., lower, and upper bounds. (Pesaran, Shin, & Smith, 2001). The hypothesis for the ARDL bounds test technique is as follows:

H_0: $\alpha_{8i} = \alpha_{9i} = \alpha_{10i} = \alpha_{11i} = \alpha_{12i} = \alpha_{13i} = 0$
H_1: $\alpha_{8i} \neq \alpha_{9i} \neq \alpha_{10i} \neq \alpha_{11i} \neq \alpha_{12i} \neq \alpha_{13i} \neq 0$

4. Results and Discussions

Descriptive statistics are employed to represent the primary elements of data inside research. The study provides concise summaries of the measurements and samples. The numerical information gathered from e-view sources is shown as charts and tables and statistically processed using E-views. The results show that the data sets variables are all adversely skewed.

Table 1: Descriptive Statistics

Variable	Observation	Mean	St. Deviation	Minimum	Maximum
unemployment rate	33	6.21	0.95	5.17	8.27
GDP	33	3.98	2.01	-0.93	7.70
FDI	33	1.09	0.82	0.37	3.66
GFCF	33	15.54	1.77	12.52	19.11
Inflation	33	9.68	7.13	0.40	38.51
Trade	33	31.97	3.95	25.30	38.49

The descriptive figures for GDP, unemployment, FDI, GFCF, inflation, and trade are explained in Table 1. The disparity demonstrates that Pakistan's unemployment is irregular and fluctuating. The unit root is crucial because this determines whether the study's series are stationary or non-stationary. Furthermore, the regression analysis is caused by a non-stationary series to be regarded as erroneous or nonsensical. The table below shows that almost all variables in this analysis are stable at a level and have a constant initial difference, trend, and constant. Most of the time series were discovered to be stationary on a trend level, according to Chaudhry, Ayyoub and Imran (2013). The results in Table 2 indicate a combination of both level and first difference, allowing us both to pursue our analysis.

Table 2: unit root tests results (ADF)

Variable	Augmented Dickey-Fuller Test Result				Decision
	At level		At first difference		
	Intercept	Trend and intercept	Intercept	Trend and intercept	
Ut	-3.258[3] 0.0277	-3.195[3] 0.1071	-4.373[0] 0.0019	-4.289[0] 0.0108	I(0)
GDP	-3.148[1] 0.0343	-2.777[0] 0.2162	-6.142[0] 0.000	-6.032[0] 0.0002	I(0)
T% GDP	-1.793[0] 0.3760	-2.319[0] 0.4108	-5.497218[0] 0.0001	-5.397[0] 0.0008	I(1)
FDI	-2.870[1] 0.0616	-2.809[1] 0.2056	-3.515240[0] 0.0150	-3.463[0] 0.0633	I(1)
GFCF	-1.706[0] 0.4173	-2.369[0] 0.3865	-4.933[0] 0.0005	-4.857[0] 0.0029	I(1)
IN	-5.049[0] 0.0003	-5.257[0] 0.0010	-8.762[0] 0.000	-8.594[0] 0.000	I(0)

The stationarity demonstrates that unemployment, economic growth and inflation are stationary on both the level and the first difference, whereas trade, FDI, and GFCF are stationary

only on the first difference. The results of the integration tests of the research variables using the Augmented Dickey Fuller test are shown in Table 2. The findings of the unit root demonstrate that the dependent variable being integrated into the level I(0). As a consequence, the study uses the ARDL Bound testing technique to estimate the study model. Unemployment has negative significant effect on GDP. The VAR regression chart depicts the various parameters for selecting dependent variable lags. Table 3 summarizes the VAR lag duration criterion for easier comprehension.

Table 3: VAR Lag order selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-321.496	NA	1382.943	24.25903	24.54699	24.34465
1	-232.079	132.470	28.43359	20.30218	22.31792	20.90156
2	-188.586	45.103	25.56069	19.74716	23.49069	20.86031
3	-98.077	53.635*	2.081*	15.709*	21.180*	17.336*

The ARDL bound testing technique was developed by Pesaran, Shin and Smith (2001) as a co-integration method to assess the long term relation between variables. ARDL bound testing method has been considered one of the preferable techniques over the other cointegration techniques. Table 4 represents the bound testing technique.

Table 4

	Model		Conclusion
	I(0)	I(1)	
Lower-upper bound (10)%	2.08	3	
Lower-upper bound (5)%	2.39	3.38	
Lower-upper bound (1)%	3.06	4.15	Co-integration
F-Statistics	10.88808		
K	5		

Table 4 displays the summary statistics of the model's Bounds test. The computed F statistics is evaluated to be (10.88), which would be greater than just the critical value Bounds connected to the upper limit of 1% significance, which is determined to be (5.761). As a result, the null hypothesis is rejected, suggesting that these two variables have such a long run co-integration relationship. The ARDL model varies amongst short and long run results, and we obtain the following findings by calculating the model's parameters. The calculated coefficient of the long term cointegration vector is shown in table 5. According to the findings, foreign direct investment seems to have a negative and significant effect on unemployment. This link is predicted on the idea that FDI reduces unemployment. According to the long run analysis, every unit increase in unemployment has a negative effect and reduces GDP growth by 0.11 units. In contrast, a one unit rise in gross fixed capital creation has a 0.96% positive significant impact on dependent variables including such Pakistan's unemployment. This study's extensive relationship verifies the findings of previous studies. Table 5 indicates the long-run coefficients.

Table 5: Long Run Results

Dependent Variable: UN				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	1.78	1.03	1.71	0.099
UN(-1)	0.96***	0.09	10.62	0.000
GDPG	-0.11***	0.03	-3.11	0.005
FDI	-0.12***	0.09	-2.90	0.008
GFCF	0.02	0.05	0.46	0.646
INF	0.03***	0.01	3.81	0.000
TRADE	-0.04**	0.02	-1.98	0.059
Diagnostics				
R-squared	0.896	F-statistic		31.64
Adjusted R-squared	0.867	Prob(F-statistic)		0.000
S.E. of regression	0.351	Durbin-Watson stat		2.457

NOTE: ***, **, and * signify coefficient significance at the 1%, 5%, and 10% levels of significance, accordingly.

Table 5 presents the long run Okun's coefficient values in Pakistan during 1991 to 2023. According to table 5, one unit rise in the GDP growth leads to a -0.11reduction in unemployment. At the 1% level of statistical significance, INF and GFCF have both positive and significant

coefficients, but FDI, GFCF, and TRADE have negative relation. Long run data show that any percentage rise in unemployment correlates to a -0.11-point drop in GDP in Pakistan.

Table 6 demonstrates short run interrelations developed from the error correcting form of the ARDL model. Using the ARDL model short run coefficients, the lagged error correction factor for the predicted GDP growth model appears to be negative with statistical significance at the 5% level of significance. In a dynamic model, the error correction term (ect(-1)) represents the rate at which changes are made to address such issues. Its estimation is -2.70, which represents a one-year reduction in variances from long-run equilibrium. Table 4.2 of the short run illustrates that one unit increase in unemployment has a (-0.16) effect on GDP growth. 1 % increase in unemployment causes inflation to 0.06%. 1 % increase in unemployment rate reduces gross fixed capital formation by -0.20%. All variables are statistically significant during the short run and have a negative effect on unemployment. Further, the residuals of the auto regressive disturbed lag model of the error correction model were verified utilizing various diagnostic tests. There is no serial correlation, heteroscedasticity, and autoregressive conditional heteroscedasticity affect in the disturbances. The Jarque-Bera normality test reveals that stochastic disturbances have such a normal distribution as well. The models are displayed in the best possible state. The ECM value in the following general statement is negative -0.70 and statistically significant 10.888.

Table 6: Short run results

Dependent Variable: UN				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.***
C	-0.10	0.05	-2.03	0.0814
DUN(-1)	1.98	0.33	5.89	0.0006
DUN(-2)	0.13	0.20	0.67	0.5204
DUN(-3)	-0.82	0.29	-2.81	0.0258
DGDPG	-0.16	0.03	-4.17	0.0042
DGDPG(-1)	0.06	0.05	1.20	0.2669
DGDPG(-2)	0.25	0.06	3.91	0.0058
DFDI	0.12	0.19	0.64	0.5381
DFDI(-1)	0.46	0.13	3.36	0.0120
DFDI(-2)	0.82	0.21	3.85	0.0063
DINF	0.06	0.01	6.32	0.0004
DINF(-1)	-0.05	0.01	-3.26	0.0137
DINF(-2)	-0.05	0.01	-3.14	0.0163
DGFCF	-0.20	0.08	-2.33	0.0519
DGFCF(-1)	-0.22	0.06	-3.39	0.0115
DGFCF(-2)	-0.17	0.07	-2.35	0.0506
DTRADE	0.02	0.02	0.85	0.4197
DTRADE(-1)	-0.10	0.03	-3.03	0.0190
ECM(-1)	-0.70	0.48	-5.61	0.0008
Diagnostics				
R-squared	0.970	F-statistic		12.915
Adjusted R-squared	0.895	Prob(F-statistic)		0.001
S.E. of regression	0.200	Durbin-Watson stat		2.034

Note: *,**, and *** signify a level of significance of 1%, 5% and 10% correspondingly.

Table 6 reveals that the ECM_{t-1} coefficient is significant and negative with a t-statistic of -5.61, confirming the long-run link among unemployment and GDP growth. The R-squared value is 0.97, and the F-statistics value 12.91, with a probability of 0.00.

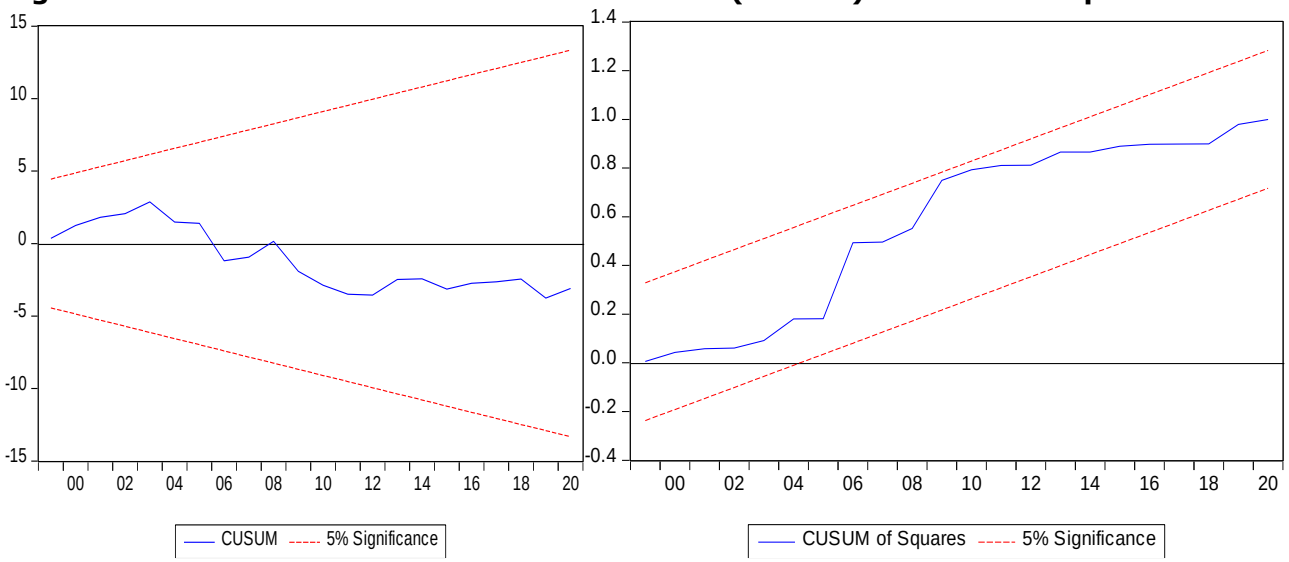
Table 7: Results of diagnostic results

"Breusch-Godfrey Serial Correlation LM Test			
F-statistic	0.2102	Prob.	0.8172
Obs*R-squared	2.0170	Prob. Chi-Square	0.3648
Heteroskedasticity Test: Breusch-Pagan-Godfrey			
F-statistic	0.3768	Prob.	0.9549
Obs*R-squared	12.795	Prob. Chi-Square	0.8036
Jarque-Bera Test of Normality			
Jarque-Bera	0.9504	Prob.	0.621741
Ramsey RESET Test"			
T-statistic	0.9278	Prob.	0.3893

F-statistic	0.8608	Prob.	0.3893

The cumulative sum of the squares of the standardized residuals demonstrates the stability of the ARDL technique. If the method of unemployment has not been changed throughout time. Fig. 1 shows that CUSUM and CUSUM of square line inside the interval bands.

Figure 1: Cumulative sum of recursive residuals (CUSUM) and CUSUM Squared



Okun's coefficient evolves with time because of rolling regression. In this study, the rolling regression coefficients are estimated using a 10-observation window, which corresponds to the span of approximately one business cycle. Figure-1 illustrates that rolling regression evaluating of the included macroeconomic variables are not constant but time fluctuating, which verifies the non-linear approach to calculating growth in equation (1). The results in table (6) indicate that GDP affects unemployment negatively and significantly at one percent level of significance. These future anticipated time-varying estimates from equation (1) for unemployment (u), (FDI), inflation (INF), Trade % of GDP, GDP growth and gross fixed capital formation as dependent variable show some fluctuation. The effect of GDP growth on unemployment is negative and significant statistically at the 5% level. The relation between GDP growth and unemployment has been stable over the period as in the previous section of the findings. But now, in this section we have described that the time varying coefficients between the GDP and unemployment the controlling this chapter is not stable. In this graph we have seen that from 2000 onward coefficients are negative after 2012 to 2019 rolling regression show the positive connection among both GDP growth and rate of unemployment. As a result, a rise in GDP doesn't lead to a reduction in unemployment. This kind of unstable relationship is not proven the Okun's law.

Figure 2: Rolling Coefficient GDPG And Unemployment

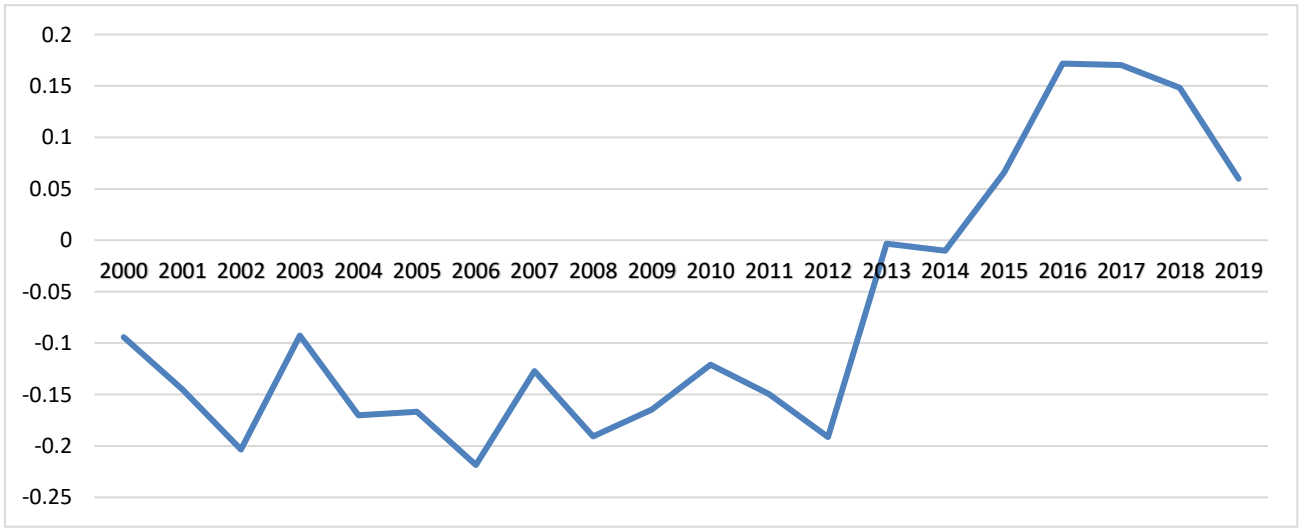


Table 8

Year	GDPG
2000	-0.09435
2001	-0.14492
2002	-0.20339
2003	-0.09265
2004	-0.17012
2005	-0.16664
2006	-0.21853
2007	-0.1273
2008	-0.19086
2009	-0.16453
2010	-0.12094
2011	-0.14954
2012	-0.19134
2013	-0.00323
2014	-0.01035
2015	0.065734
2016	0.171724
2017	0.170195
2018	0.14809
2019	0.059846

The findings of the time-varying coefficient study for Pakistan are summarized in figure 1. Figure 1, which presents the findings for the period of observation 1991 to 2020, confirms that the Okun's coefficient of Pakistan researched here varies throughout time. Rolling regression indicates that the Okun's coefficient fluctuates with time. According to rolling regression unemployment coefficient shows fluctuations and changes over time. GDP coefficient shows small ups and down in magnitude of the Okun's coefficient until 2014, following which it turns upward. Gross fixed capital formation demonstrated the highest Okun's coefficient when compared to other coefficients. Trade exhibit increasing than decreasing trend during the period 2010 to 2020. The projected rise in unemployment for a given reduction is small, according to the Okun's coefficient calculated using rolling regression. Okun's law dynamics are the opposite of rolling regression.

Figure 3: Rolling regression coefficients for each variable in the model

Source: the authors construct rolling regression estimates for the variables

5. Conclusion

The research examines the calculation of Okun's Law with time varying coefficients and test the existence of Okun's law in the long run in Pakistan. Rolling regression was employed to calculate the coefficient of Okun's law. Time-series data from Pakistan Economic Survey between 1991-2023 has been gathered for this objective. The rate of unemployment has been employed as a dependent variable in this study. As independent variables, FDI, gross fixed capital creation, inflation GDP, and trade all are taken into consideration. This study examines Okun's law in Pakistan utilizing the autoregressive distributive lag bounds testing technique. The ARDL findings show that the variables have cointegration, demonstrating both long and short-term relationship between them, though the change coefficient varies over time. The ECM value in short-run estimation is a negative -2.70. This study finds a long-term detrimental link between trade. The study concludes that GDP Growth and unemployment has significant long run association in Pakistan. Okun's Law exist in Pakistan and diagnostic tests has showed the stable findings in the long run and short run. Time varying coefficients are not stable over the period of analysis. There is need to consider other factors such as FDI, and trade accompanying with GDP growth to reduce unemployment in Pakistan. The study demonstrates that the GDP growth necessary to sustain the long-term rate of unemployment has been reduced significantly lowered over the past ten years by evaluating a theory with time-varying components. It could also be stated thus Pakistan's unemployment problem is not caused by a shortage of growth. Additionally, Ramsey reset, Breusch Godfrey Serial Correlation LM test and White Heteroskedasticity outcomes are used to discover the co-integrating component and regression analysis with the use of econometric software. According to E-Views, there are no issues with serial correlation, model misspecification, or heteroskedasticity. A normal distribution for stochastic disturbances is also suggested by the Jarque-Bera test for normalcy. According to CUMUS and CUMUMSQ the model is sustainable within 5% of critical limits. This study leaves certain concerns unsolved, and it will inspire future researchers to apply other versions of Okun's law to larger time series and other states of observations. This will ensure that more effective results are achieved. Our findings continue to be beneficial to policy makers who, to some measure, consider macroeconomics connection of this form when making choices.

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