



## **Quantifying the Impact of Measurement Errors in Consistent Linear Partial Least Squares Structural Equation Modelling: A Monte Carlo Investigation**

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### **ABSTRACT**

Measurement errors wield the potential to introduce uncertainties and inaccuracies, casting shadows on data quality and jeopardizing the integrity of structural relationships. Notably robust against measurement errors, Partial Least Squares Structural Equation Modelling (PLS-SEM) has historically maintained a reputation for resilience. However, recent insights have unveiled its susceptibility to these errors, instigating a reevaluation of its standing in the Structural Equation Modelling landscape. Overlooking measurement errors in PLS-SEM carry consequential repercussions, notably tainting the accuracy of structural relationships and introducing bias. This effect becomes particularly pronounced when dealing with an insufficient understanding of the intricate structural dynamics. Unfortunately, PLS-SEM currently lacks an all-encompassing remedy to address this concern. Consequently, the quantification of measurement errors impact in PLS-SEM gains paramount importance, fostering a demand for innovative strategies to propel its effectiveness forward. Notably, contemporary investigations have unmasked PLS-SEM's vulnerability to non-orthogonal errors. This revelation challenges the notion of its imperviousness to the detrimental influence of measurement errors, necessitating a comprehensive evaluation of its performance under such conditions. This study leveraged simulated data to extract empirical findings and employed parameters biasedness analysis. This analysis led to the determination that the stability of the PLS-SEM algorithm is compromised when exposed to diverse measurement error scenarios. Consequently, the outcomes generated exhibit both instability and bias. This bias becomes increasingly conspicuous as the magnitude of measurement errors intensifies. Thus, the study proposes avenues for elevating the robustness of PLS-SEM.



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## **1. Introduction**

Structural Equation Models (SEMs) have emerged as a diverse set of statistical methodologies aiming to estimate causal relationships within theoretical models (Bollen, 1989; Kaplan, 2008). These models act as a link between latent complex concepts, each measured by a set of observable indicators. The fundamental premise behind SEMs is to analyse the intricacies within a system by exploring the networks of causality among latent variables, where each variable is assessed through multiple observed indicators, often referred to as Manifest Variables. This unique approach allows researchers to delve into the complexity of relationships between latent constructs and their observable manifestations (Hamblin & Hauser, 1975; Luce & Tukey, 1964). Consequently, SEMs serve as a convergence point between Path Analysis and Confirmatory Factor Analysis, combining the causal modelling aspects of the former with the measurement modelling principles of the latter (Thurstone, 1961).

Two distinct conceptual approaches have been proposed for analysing Structural Equation Models (SEMs): the factor-based SEM Jöreskog (2007) and the composite-based SEM (Wold, 1975; Wold, 1982).

In the factor-based SEM, also known as CB-SEM (Common Factor-based SEM), the unobservable or conceptual variables are approximated by common factors. This approach assumes that the construct exists independently of the observable variables and is the primary source of variation among them. In essence, factor-based SEM focuses solely on capturing the shared variance among latent and observable variables. Conversely, the composite-based SEM, also known as Partial Least Squares Path Modelling (PLSPM), represents the construct as a weighted composite or a combination of observed variables. This approach considers the construct as an aggregation of observable variables. It relies on traditional multivariate techniques like Principal Component Analysis (PCA) and Canonical Correlation Analysis (CCA), setting it apart from its factor-based counterpart (Arrow & Lehmann, 2006; Godshalk & Timothy, 1988; Horst, 1965; Pearson, 1901).

These methodological distinctions can be traced back to the works of Jöreskog and Wold (1982) respectively. The factor-based SEM draws inspiration from the classic works of Horst, Hotelling, Pearson, and Spearman, dating back to the early 20<sup>th</sup> century. On the other hand, the composite-based SEM builds upon established multivariate techniques, emphasizing the utilization of PCA (principal component analysis) and CCA (canonical correlation analysis) to elucidate relationships among observed variables.

The PLS approach to structural equation modelling has been proposed as the component-based estimation procedure different from the covariance based structural equation modelling. The main principals of the partial least squares were discussed in a seminal paper by Wold (1966) were extended to the more than one block of variable. Since its inception several extensions have been proposed to issues arising in the PLS-SEM procedures. The most common extensions include the confirmatory tetrad analysis (CTA- PLS), importance performance matrix analysis (IPMA), multi group analysis (MGA-PLS) and various approaches to assess the hierarchical component models, analysis of various interactions effects and the most important consistent partial least squares (PLSc) and various approaches to treat the correlated measurement errors. The recent advancements in the PLS-SEM include the various criterion for assessing discriminant validity based on the HTMT ratio of common factor correlation Henseler, Ringle, and Sarstedt (2015), utilization of the standardized root mean square residual (bootstrap-based) as an indicator for overall model fit Dijkstra and Henseler (2015a), incorporation of the Consistent Partial Least Squares SEM Dijkstra and Henseler (2015a, 2015b) to mitigate PLS bias, and adoption of Ord-PLS-SEM for handling ordinal data (Schamberger, Schuberth, Henseler, & Dijkstra, 2020).

The significance of these advanced techniques and PLS algorithm extensions is immense, aiding researchers and scientists in attaining the minimum estimation and analysis standards like other multivariate methodologies. The methodological progress and numerous extensions in PLS-SEM underscore the triumph of partial least squares structural equation modelling, particularly in domains characterized by limited sample size and less restrictive distributional assumptions (Reinartz, Haenlein, & Henseler, 2009). However, it's important to note that utilizing PLS as an estimator for structural equation modelling is not exempt from drawbacks and is robust against measurement errors.

Despite the elegant and flexible modelling characteristics of PLS-SEM, several substantial concerns necessitate attention. A prominent issue pertains to estimating structural models in scenarios featuring measurement errors and non-orthogonal manifest variables or indicators. No existing simulation study has yet explored whether substantial measurement errors within structural models and indicators impact the adequacy and effectiveness of path coefficient estimations in variance-based SEM (Dijkstra & Henseler, 2015b).

Neglecting measurement errors can yield far-reaching consequences for modelling approaches in variance-based structural equation modelling (Rigdon, 2014). Variance-based SEM has the potential to inflate path coefficients and magnify "t" values if these errors are disregarded (Goodhue, Lewis, & Thompson, 2012; Henseler, Hubona, & Ray, 2016). Consequently, the limitations of the Consistent Partial Least Squares estimator (PLSc) also become evident when dealing with potential measurement error issues among structural variables and indicators. This can result in relatively reduced statistical power and larger standard deviations (Dijkstra & Henseler, 2015b). This tendency could be more pronounced, particularly in cases of estimations involving small sample sizes, contradicting the fundamental design of the PLS-SEM algorithm.

Measurement errors means inaccuracies and uncertainties introduced during the process of data collection or measurement. These errors can affect the reliability and validity of the data and structural relationships. The Partial least squares structural equation modelling (PLS-SEM) which is known to be the robust method against the measurement errors is no free from such fatal errors in data and not as soft as described in the literature of the SEM based on the data properties. Thus, the non-orthogonal measurement errors if ignored may have profound impacts on the performance of the model and the structural relationships may also be biased as do the measurement relationships. The variance based structural equation modelling is meant where we do not have much knowledge of the structural relationships and the purpose is to test the theory not to confirm the theory. Therefore, in the situation where we do not have much knowledge of the structural relationships the measurement errors have broad implications. The PLS-SEM do not have remedy to account for such measurement errors. Furthermore, the theory of measurement states that the formative models are assumed to be error free in a conventional sense, these assumptions have broader implications in the case where the variables are non-orthogonal and the evaluation of models involving formative indicators becomes complicated Hair, Ringle, and Sarstedt (2013) and the consequences of such errors are an issue in such situation. Therefore, the quantification of the impact of the measurement errors in case of PLS-SEM is more important. Keeping in view the above discussion, therefore, the major purpose of the current paper is to empirically quantify the impact of the measurement errors non-orthogonality and assess its impacts on the parameters of the model, on the structural relationships and suggesting the way forward for empirical advancements.

As for the significance of the study is concerned, this paper will be a first attempt to explicitly measure the impacts of correlated measurement errors on the parameter estimates of the PLS-SEM framework. The renowned consistency (attenuation corrected) constant which is widely used in the PLS-SEM to account for the errors arising from the deviation of basic design is widely used to correct the inconsistency issue (Dijkstra & Henseler, 2015a). But that is another form of inconsistency (arising due to deviations from the basic design). Given the correlated errors and non-orthogonal nature of the variables in case of social sciences, it is not possible to

fulfil the restricted assumption orthogonality of the systematic errors of the variables of interest. So, this paper describes in detail the impact of non-orthogonal measurement errors on the measurement models and then on the structural parameters. This has wider implications as when there is no developed theory and assumption lies to develop (exploratory nature) a theory. The magnitudes and signs of the structural coefficients may have profound impacts, which are ignored relying on the assumption of the strict orthogonality. Therefore, the study will contribute to the PLS-SEM literature on sensitizing the role of correlated measurement errors and their consequences on theory development.

The rest of the paper is organized in three sections. In the first section existing literature is analyzed and in remaining two sections empirical estimations and data analysis is presented.

## **2. Literature Review**

The CB-SEM and PLS-SEM were developed almost at the same time in 1980's. The CB-SEM became more widely used in operational research because of its early availability through the LISEREL software since the late 1970's. In contrast the first software of the PLS-SEM was the LVPLS Jagpal (1982) it was not a much user friendly later, PLS- GRPAH was developed by Chin and Todd (1995) and PLS-SEM got its momentum in various applications. With the development of Smart- PLS Ringle, Wende, and Will (2009) PLS-SEM applications were grown exponentially. So, the period from 1980 to 2000, it could not get many modifications and advancements as did the covariance-based structural equation modelling. After the development of Smart-PLS there got the era of modifications and advancements in the PLS-SEM.

Vinzi, Chin, Henseler, and Wang (2009) introduced the methods to capture the unobserved heterogeneity in PLS-SEM the method commonly known as response-based unit segmentation analysis (REBUS- PLS) which generally relocates the observations from one segmentation to another while minimizing the residuals of the model. There were several other approaches to deal with the unobserved heterogeneity which were introduced in the same year. Notably, it includes renowned work of (Aluja-Banet & Sánchez; Hahn, Johnson, Herrmann, & Huber, 2002; Ringle et al., 2009; Vinzi, Ringle, Squillacciotti, & Trinchera, 2007).

Henseler, Ringle, and Sinkovics (2009) presented the comparisons of four approaches to study the interaction effects between the latent variables using the partial least squares structural equation mode lining. They compared the product indicator approach Chin, Marcolin, and Newsted (2003), the two stage least squares approach Chin et al. (2003), a hybrid approach by Wold (1982) and an orthogonalizing approach Little and Rubin (2019) in terms of the accuracy of the point estimate, statistical power and prediction accuracy in the PLS-SEM framework by using extensive simulation and they recommended the orthogonalizing approach is better one where even the sample size is small.

The hierarchical components model has long history in the covariance based structural equation modelling. Streukens, Wetzels, Daryanto, and de Ruyter (2010) introduced the concept into the PLS-SEM framework and further it was extended by Ringle, Sarstedt, and Straub (2012) who implemented this idea empirically in the information systems. Chin and Dibbern (2009) introduced the distribution free approach to the Multi group analysis in the PLS-SEM (MGA- PLS). it was further empirically implemented by Ringle et al. (2009). In the same decades, numerous articles were published on the observed heterogeneity in the PLS-SEM framework. Notable work in the subject includes the Finite mixture PLS by Ringle et al. (2009) the prediction-oriented segmentation analysis by Becker, Rai, Ringle, and Völckner (2013) and the genetic segmentation analysis (Ringle et al., 2009). Nowadays, to assess the observed heterogeneity in the PLS-SEM multi group analysis approach by Sarstedt (2019) is commonly applied. Other advance in the pls-sem include the introduction of the quadratic effects of the formative indicators by Henseler et al. (2016) and various approaches to discuss the consistency of the measurement and

structural models given the basic design of the PLS-SEM. Notably, includes the (Gefen, Rigdon, & Straub, 2011; Goodhue et al., 2012; Henseler et al., 2014; Raykov & Marcoulides, 2012).

Dijkstra and Henseler (2015a) introduced the consistent partial least squares algorithm (PLSc) to account for the inconsistency arising from the deviations of the assumption of the basic design that uses the formative and reflective measurement indicators simultaneously and they also concluded that the bias may also be present in the estimates if the measurement errors are present in the model. Therefore, they concluded that pls-Sem can be made equally competing with the CB- SEM even when the formative indicators are used in the PLS-SEM model using the consistent approach (PLSc) and that the measurement errors affect the size of the bias of the model.

Chust, Steinle-Neumann, Dolejš, Schuberth, and Bunge (2017) introduced the Ord-PLS to account for the ordinal data and in the same year Henseler et al. (2016) introduced the three-step procedure to account for the measurement invariance. Hair Jr and Sarstedt (2019) used the discrete choice models to study the PLS-SEM most used models were logit, probit, and multinomial logit and thus suggested the situations in which how the PLS-SEM can be used with the binary ordinal data to model the relationships. Schamberger et al. (2020) conducted a simulation study to draw conclusions about the effects of data distortions on the parameter estimates of the PLS-SEM. The literature on how the measurement errors affect the model accuracy and consistency in the PLS-SEM framework is still not much available. Therefore numerous ad hoc approaches on how to deal with the measurement errors if these are present in the model but there is no scientific approach as do in the econometric framework to account these measures.

There are two extremes to account for measurement errors correlation in the pls-sem on is the approach by Rademaker (2020) which is to define the set of non-orthogonal variables and excluding them from the analysis which is itself a specification error. While other extreme is to go with these errors as structural equation models are robust against the measurement errors which is general phenomenon without any scientific base. While some have shown serious concerns Jagpal (1982) So, the question of how to address the measurement errors in measurement model and/or in the structural models is still unaddressed and the available solutions are of limited nature. Some proposals the met in the literature are of deleting the correlated indicators when betrayal signs of the orthogonal appear and that the generating a new variable using the principal component analysis Hair et al. (2013) but, these approaches cannot be taken as for granted in a case where the application of method is when do not have much knowledge of the theory and we are not confirming but developing a theory.

### **3. Methodology and Data-Generating Process**

The PLS-SEM methodology consists of estimating the two models simultaneously: the outer model or the measurement model and the inner model or the structural model. The convergence is achieved by an iterative procedure based on a certain truncation level. The convergence procedure comprised of four steps: outer approximation of latent variable scores, estimation of the inner weights, inner approximation of latent variable scores, and finally estimation of the outer weights. PLS-SEM is a non-parametric approach to structural equation modelling therefore, the bootstrap procedure is opted for the tests of goodness of fit of the model. The outer model or the measurement model is framed based on two theories that is the theories of measurement. The formative approach and the reflective approach. The choice of each theory of measurement is based on the underlying theory and objective of the study. The mathematics behind the PLS-SEM may be summarized as follows:

If we let us assume the model with  $J$  latent constructs  $\eta_1, \eta_2, \dots, \eta_j$  contained in a  $J \times 1$  vector and connected by the structural model. The constructs are either modelled as the reflective way

or the formative way. There is  $K \times 1$  vector of the indicator variables denoted as:  $x_1, x_2, \dots, x_j$  defined as the measurement errors prone manifestation of their respective constructs. Then the model can be represented as follows.

$$x_j = \gamma_j \eta_j + e_j \tag{1}$$

Where,  $j$  is  $j = 1, 2, 3, \dots, J$

The measurement errors are assumed to possess the following two properties.

- i.  $E(e_j | \eta_j) = 0$ , which means that the conditional mean of the  $x_j$  is given by  $E(x_j) = E(\gamma_j \eta_j) + E(e_j)$  and which further simplifies to  $E(x_j) = (\gamma_j \eta_j)$  and
- ii.  $E(\epsilon_i \epsilon_j) = 0$

These properties are the results of classical assumptions of on the conditional mean and spherical distribution of the errors, in view of the above, the measurement errors correlation between the blocks is given by  $E(\epsilon_i \epsilon_j) = \sigma^2$  and across the blocks is  $E(\epsilon_i \epsilon_j) = 0$ . Based on these the following variance covariance matrices are formulated.

$$\Sigma_{ij} \equiv E(x_i x_j) = \rho_{ij} \lambda_i \lambda_j \tag{2}$$

$$\Sigma_{ij} \equiv E(x_i x_j) = \rho_{ij} \lambda_i \lambda_j \tag{3}$$

$$\Sigma_{jj} \equiv E(x_j x_j) = \lambda_j \lambda_j + \sigma^2 \tag{4}$$

If we let us assume the sample size of 'n' and for simplicity and no loss of generality, if we group the indicators belonging to one common factor or composite together to form a block 'j' and the observations of the block  $J$  are stacked in the data matrix  $X_j$  of the dimension  $(n \times K_j)$  with the restriction that  $\sum_{j=1}^J k_j = k$  and it is assumed that each block of observed variable is standardized then all the variables of the  $k_j$  indicators are stacked in a data matrix  $X$ . It is important to note that the initialization of PLS algorithm takes place by assigning the arbitrary weights in such a way that  $\hat{w}_j^{(0)} S_{jj} \hat{w}_j^{(0)} = 1$ . There are several ways to assign weights but most used are path weighting scheme, factorial weighting scheme the centroid weighting scheme. In this way the algorithm initiates and four steps are completed.

There are two ways in which the relationships in the measurement level can be accommodate. The mode 'A' in which it is assumed that the constructs exist, and this is the whole source of variation in the indicator variables. Therefore, there are as many regressions as many relationships are described in the path diagram. If we assume the regression of full data matrix of indicators and the latent variables, then following proportional relationship can be equivalently written as.

$$\hat{w}_j^{(h+1)} \propto \sum_{i=1}^j e_{ji}^{(h)} S_{ij} \hat{w}_i^{(h)} \quad \text{with,} \quad \hat{w}_j^{(h+1)} S_{jj} \hat{w}_j^{(h+1)} = 1 \tag{5}$$

For mode 'B' we know that there are single multiple regression equations, and the weights are the regression weights therefore, using the notation above the proportional relationship can be written as

$$\hat{w}_j^{(h+1)} \propto (S_{jj})^{-1} \sum_{i=1}^j S_{ji} \hat{w}_i^{(h)} e_{ji}^{(h)} \quad \text{with,} \quad \hat{w}_j^{(h)} S_{jj} \hat{w}_j^{(h)} = 1 \tag{6}$$

In this way, the iterative algorithm approaches to the convergence and the final weights are used to build the latent variables, factor loadings and the path coefficients are the Ordinary least squares solution of the equations postulated by the structural model. If we let us

assume that the path models are recursive then the path coefficients are obtained by using the well-known results of the matrix algebra in the theory of regression analysis as:

$$\beta_{(pls-sem)} = (R_X)^{-1} \cdot r_{xy} \quad (7)$$

$R_X$  is the correlation matrix of the independent variables and  $r_{xy}$  is the correlation between the independent and dependent variables. The assumption on the basic design of the PLS-SEM describes that the process is based on common factors but when the constructs are modelled as composite way then there is inconsistency in the parameter estimates of the outer model and the inner model and this inconsistency cannot be avoided until the measurement errors are zero in the model. This deviation from the basic design and the issue of inconsistency was addressed by Dijkstra and Henseler (2015a) in his famous proportionality constant to the population loadings. This proportionality constant is given by  $c_j^2 = \lambda_j' \Sigma_{jj} \lambda_j$ . This is also known as the correction for attenuation.

This correction for attenuation is achieved in such a way that the squared Euclidean distance between the off-diagonal elements of the empirical covariance matrix  $S_{jj}$  and the matrix implied by the composite indicators  $(C_j \hat{w}_j)(C_j \hat{w}_j)'$  is minimized. Thus, utilizing the above information the correction for attenuation written in the form of matrix notation as above can be written as.

$$\hat{c}_j^2 = \frac{\hat{w}_j'(S_{jj} - \text{diag}(S_{jj}))\hat{w}_j}{\hat{w}_j'(\hat{w}_j \hat{w}_j' - \text{diag}(\hat{w}_j \hat{w}_j'))\hat{w}_j} \quad (8)$$

It is important to note here that this correction for attenuation is based on the strict assumption that the measurement errors in the above expression are zero and that the system follows the assumption of basic design. When fulfilling these assumptions, the numerator in the above equation becomes a null matrix and the correction for attenuation becomes  $\hat{c}_j^2 = \lambda_j' \Sigma_{jj} \lambda_j$ . this is the squared distortion of the population weights to the population loadings. Hence, the consistent factor loadings estimate, and the attenuation corrected correlation between the common factors  $j$  and  $i$  are given by:

$$\hat{\lambda}_j = \hat{C}_j \hat{w}_j \text{ and } \text{corr}(\hat{\eta}_j, \hat{\eta}_i) = \frac{\hat{w}_j' S_{ij} \hat{w}_i}{\sqrt{\hat{\rho}_{Aj} \cdot \hat{\rho}_{Ai}}} \quad (9)$$

This shows the estimated deattenuated correlation. The consistent path coefficients of the underlying structural model are estimated using the ordinary least squares regression or the two stage least squares regression, that is by using.

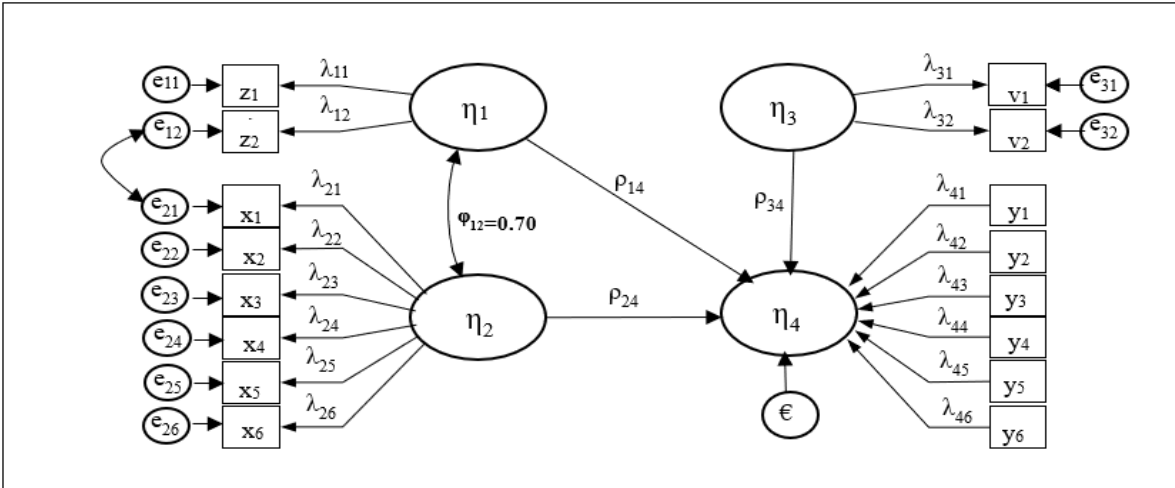
$$\hat{\beta}_c = (R_X)^{-1} \cdot r_{xy} \quad (10)$$

$\hat{\beta}_c$  are now the consistent estimates of the path coefficients. It is interesting to note here that deviation from the basic design and assumptions is always costly and this cost appeared in the form biased parameter estimates. This piece of paper is an effort to account for these biases and suggesting a way forward.

### 3.1. A Simulation Study: Data Generating Process and Design Factor

To capture the effect of measurement error non orthogonality. A simulation study base on various experimentally manipulated conditions is conducted and the base line model for simulation is chosen in such a way that it resembles the commonly used models in the applied research (Chen, Bollen, Paxton, Curran, & Kirby, 2001). The simulation model consists of four latent variables selected on the findings of Shah and Goldstein (2006) guidelines using the

commonly used configuration used in the PLS-SEM. The Montecarlo simulation examines a scenario with three exogenous latent variables and one endogenous. The exogenous are again measured by the maximum of six indicators denoting mode A for laten variable  $\eta_2$  the latent variable  $\eta_4$  with the six formative indicators and other two with the two indicators each. The following diagrams shows the specific data generating process:



**Figure 1: Population model**

The current problem of quantification of measurement errors is to quantify the impact of the measurement errors in the PLS-SEM framework. The measurement errors are thus incorporated into the model through the non-orthogonal errors of the model at measurement level. This is done by varying the strength of the association between the errors and across the indicators as shown in the figure above. The association is varied from 0.1 to 0.9 and its impact on the true or population parameters of the model is observed. The true or population parameters are also defined by the following way. If we just assume keeping the strength of the model accurate and test for goodness of fit as minimum acceptable then, the model parameters are said to have the following population values.

The factor loadings are fixed at  $\lambda_{11} = .65$ ,  $\lambda_{12} = .70$ ,  $\lambda_{21} = .75$ ,  $\lambda_{22} = .80$ ,  $\lambda_{23} = .65$ ,  $\lambda_{24} = 0.72$ ,  $\lambda_{25} = .75$ ,  $\lambda_{26} = .78$ . Similarly for the formative construct the following is assumed  $\lambda_{41} = .65$ ,  $\lambda_{42} = .80$ ,  $\lambda_{43} = .75$ ,  $\lambda_{44} = .80$ ,  $\lambda_{45} = .65$  and  $\lambda_{46} = .82$  for the latent variable  $\eta_4$  we have the following  $\lambda_{31} = .70$  and  $\lambda_{32} = .75$ . This constitutes in other words the population measurement model. The structural model or inner model is formulated as:  $\rho_{14} = .17$ ,  $\rho_{24} = .56$  and  $\rho_{34} = .70$ . The measurement errors orthogonality as shown by the population model above is incorporated by varying the strength of correlation between and across the errors and the corelation between the indicator variable is also observed. The ranges for correlation are like that of Grewal, Cote, and Baumgartner (2004). Thus, under settings given above the model is assumed to behave following.

- i. The effect of  $\eta_1$  on  $\eta_4$  ( $\rho_{14} = .17$ ) which is not expected to be affected by the measurement errors correlation and the non-orthogonal indicator variables. It is expected that the PLSc in this case will recover the biased parameters. The direction of bias may be in either direction.
- ii. Similarly, the effect of  $\eta_2$  on  $\eta_4$  ( $\rho_{14} = .56$ ) which is assumed not to be substantially affected by the non-orthogonal latent variables and by the measurement error correlation. The PLSc in this case is assumed to produce the biased estimates in both the models that is in measurement models and in the structural relationships.



- iii. The effect of  $\eta_3$  on  $\eta_4$  ( $\rho_{34} = .70$ ) here, there are measurement errors correlation in both the models that is reflective and formative therefore, it is expected to behave not erroneously both in the measurement models and in the structural relationships as well.

Given the simulations designs and design factors the above data generating process is connected by the following simultaneous equation systems. The structural relationships are given by,

$$\eta_4 = \alpha + \theta_1\eta_1 + \theta_2\eta_2 + \theta_3\eta_3 + \varepsilon \quad (11)$$

Where the measurement models are linked with the following equations:

$$z_1 = \alpha + \lambda_{11}\eta_1 + v_1 \quad (12)$$

$$z_1 = \alpha + \lambda_{11}\eta_1 + v_1 \quad (13)$$

$$z_2 = \alpha + \lambda_{12}\eta_1 + v_2 \quad (14)$$

$$x_1 = \alpha + \lambda_{21}\eta_2 + e_1 \quad (15)$$

$$x_2 = \alpha + \lambda_{22}\eta_2 + e_2 \quad (16)$$

$$x_3 = \alpha + \lambda_{23}\eta_2 + e_3 \quad (17)$$

$$x_4 = \alpha + \lambda_{24}\eta_2 + e_4 \quad (18)$$

$$x_5 = \alpha + \lambda_{21}\eta_2 + e_5 \quad (19)$$

$$x_6 = \alpha + \lambda_{26}\eta_2 + e_6 \quad (20)$$

$$v_1 = \alpha + \lambda_{31}\eta_3 + \xi_3 \quad (21)$$

$$v_2 = \alpha + \lambda_{33}\eta_3 + \xi_3 \quad (22)$$

And  $\eta_4$  is measured by formative indicators,

$$\eta_4 = \alpha + \lambda_{41}y_1 + \lambda_{42}y_2 + \lambda_{43}y_3 + \lambda_{44}y_4 + \lambda_{45}y_5 + \lambda_{46}y_6 + \varpi \quad (23)$$

Although there are many cases in which measurement errors can enter the model, by the present study assumes the following two scenarios.

1. Non-Orthogonal Measurement errors across the indicator variables
2. Non-Orthogonal latent variables due to Measurement errors

Keeping in view the above setting of the population model, random numbers are drawn from the multivariate normal distributions with the sample size of 400. This is replicated 50,000 times to draw the conclusions. The study is based on relaxing the assumption of  $E(\epsilon_i \epsilon_j) = 0$ . Therefore, varying the correlation between and across the indicators and latent variables and each time calculating the model for fifty thousand time the results are portrayed. The simulation is conducted in the R environment with the LAVAN and SEMinR Package, the next section summarizes the simulation results and possible findings of the study.

#### 4. Simulation Results

This section comprises of detailed results of the simulation experiment which describes the ability of the PLSc to recover the true parameter values of the structural model as well the measurement model. The practical significance of Partial Least Squares (PLSc) in real-world scenarios might rely on its capacity to ascertain the importance of a parameter estimate from the standpoint of statistical power. While accurate statistical inferences are crucial for conducting dependable hypothesis tests, it is equally important to assign similar importance to the size of the structural parameters for result interpretation in predictive contexts. Hence, assessing the capability to accurately retrieve true parameters holds significance for practical researchers contemplating the adoption of PLS-SEM method.

To assess the recovery of the Parameter under the given experimental settings, the mean absolute deviation (MAD) between the true parameter and their estimates is calculated as follows:

$$MAD = \frac{\sum_1^p |\hat{\theta}_j - \theta_j|}{p} \tag{24}$$

**Table 1**  
**Results of the Simulation**

**Scenario 1: Non-Orthogonal Measurement errors across the indicator variables**

Measurement Model		weights															Structural Model Path Coefficients			
PLSc	φ	Loadings						weights									p <sub>14</sub>	p <sub>24</sub>	p <sub>34</sub>	
		λ <sub>11</sub>	λ <sub>12</sub>	λ <sub>21</sub>	λ <sub>22</sub>	λ <sub>23</sub>	λ <sub>24</sub>	λ <sub>25</sub>	λ <sub>26</sub>	λ <sub>31</sub>	λ <sub>32</sub>	λ <sub>41</sub>	λ <sub>42</sub>	λ <sub>43</sub>	λ <sub>44</sub>	λ <sub>45</sub>				λ <sub>46</sub>
0.	0.6	0.6	0.7	0.6	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.700
1	50	00	78	00	50	20	50	80	00	50	50	00	50	00	50	20	70	60		0.700
0.	0.6	0.7	0.6	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.700	
2	50	00	84	00	50	20	50	80	00	50	50	00	50	00	50	20	70	60		0.700
0.	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.700	
3	55	05	11	01	50	21	50	80	00	50	51	01	50	05	53	22	69	60		0.698
0.	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.698	
4	61	10	27	07	51	25	55	81	01	50	52	04	50	05	53	22	69	12		0.698
0.	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.698	
5	63	15	83	10	52	28	55	82	04	51	52	09	51	07	54	25	64	12		0.670
0.	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.670	
6	75	16	81	17	51	31	52	83	02	52	51	10	51	08	58	29	62	01		0.670
0.	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.670	
7	78	23	86	21	53	34	58	83	04	51	53	10	54	09	57	29	62	01		0.661
0.	0.7	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.4	0.661	
8	00	36	89	21	55	36	58	85	03	53	57	15	55	09	78	30	60	93		0.657
0.	0.7	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.4	0.657	
9	50	44	93	22	61	38	59	89	04	51	59	14	54	11	79	30	57	91		0.70
Pop. Parameter	0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.1	0.5	0.70	

Sample size N= 400, Number of Simulations R= 50,000 and Phi (φ) is the strength of association between the measurement errors.

**Table 2**  
**Biasedness Analysis (MAD)**

**Scenario 1: Non-Orthogonal Measurement errors across the indicator variables**

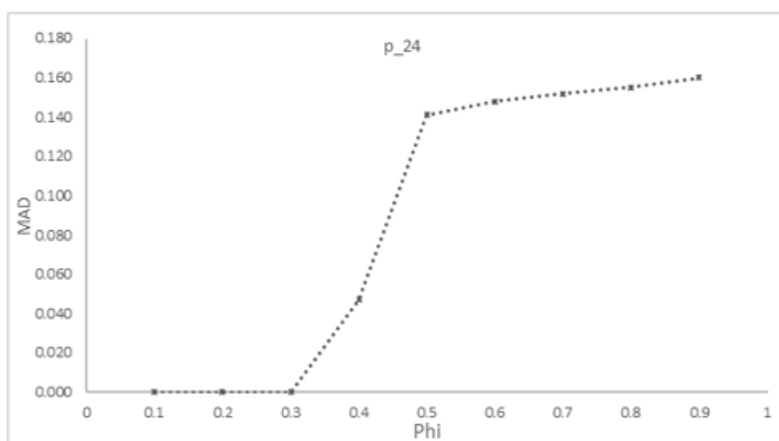
Measurement Model		weights															Structural Model Path Coefficients			
PLSc	φ	Loadings						weights									p <sub>14</sub>	p <sub>24</sub>	p <sub>34</sub>	
		λ <sub>11</sub>	λ <sub>12</sub>	λ <sub>21</sub>	λ <sub>22</sub>	λ <sub>23</sub>	λ <sub>24</sub>	λ <sub>25</sub>	λ <sub>26</sub>	λ <sub>31</sub>	λ <sub>32</sub>	λ <sub>41</sub>	λ <sub>42</sub>	λ <sub>43</sub>	λ <sub>44</sub>	λ <sub>45</sub>				λ <sub>46</sub>
0.1	0.00	0.0	0.0	0.07	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.00	0.00	
	0	00	2	00	00	00	00	00	00	00	00	00	00	00	0	00	0	0	00	0
0.2	0.00	0.0	0.06	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.00	0.00	
	0	00	6	00	00	00	00	00	00	00	00	00	00	0	00	0	0	00	00	0
0.3	0.00	0.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.00	0.00	0.00	0.00	
	5	05	9	01	00	01	00	00	00	00	01	01	00	5	03	2	1	00	0	
0.4	0.01	0.0	0.02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.00	0.00	0.00	0.00	
	1	10	3	07	01	05	05	01	01	00	02	04	00	5	03	2	1	48	2	
0.5	0.01	0.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.00	0.00	0.00	0.00	
	3	15	3	10	02	08	05	02	04	01	02	09	01	7	04	5	6	48	2	
0.6	0.02	0.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.00	0.00	0.00	0.03	
	5	16	1	17	01	11	02	03	02	02	01	10	01	8	08	9	8	59	0	
0.7	0.02	0.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.00	0.00	0.00	0.03	
	8	23	6	21	03	14	08	03	04	01	03	10	04	9	07	9	8	59	0	
0.8	0.05	0.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.01	0.01	0.0	0.03	
	0	36	9	21	05	16	08	05	03	03	07	15	05	9	28	0	0	67	9	
0.9	0.10	0.0	0.04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.0	0.01	0.01	0.0	0.04	
	0	44	3	22	11	18	09	09	04	01	09	14	04	1	29	0	3	69	3	

By utilizing MAD as a measure of performance, this study aims to distil the essence of technique effectiveness and shed light on their relative merits. The subsequent synthesis encapsulates the essence of the simulation's findings, offering a succinct overview of the techniques' performance in relation to MAD.

The outcomes of the simulation study reveal significant insights into the performance of the analysed techniques, with mean absolute deviation (MAD) serving as the key yardstick for assessing their efficacy. Through rigorous analysis, it becomes evident that the techniques' performance varies in relation to the magnitude of MAD values. Techniques exhibiting lower MAD values showcase superior accuracy and precision in approximating the desired outcomes. Conversely, higher MAD values are indicative of greater discrepancies between estimated and actual values, highlighting potential limitations in certain approaches. These results underscore the critical role of MAD as a reliable indicator for evaluating the robustness and reliability of the techniques under scrutiny.

The parameter estimates under varying conditions and of the measurement models and the structural. The table 1 represents the parameter estimates of the simulated data with the true parameter values given in the last row of the table. It is clear from the table that the outer model or measurement model is overestimated in this case when there are measurement errors correlation across the indicator of the model.

Therefore, the parameter estimates are biased upward, and the strength of biasedness increases as the error's correlation increases. Similar effects have been observed in the case of formative model. The structural parameters are biased downward. As the strength of correlation across the measurement errors increases the structural parameter falls away from the true parameter. Therefore, the parameter estimates of the structural model are biased downwards. This inconsistency is like that of the findings of Henseler et al. (2015) when deriving the PLSc in case when there are reflective and formative model associated in the path diagram. The absolute bias calculated from the given results is presented in the table 2 under the same case of non-orthogonality.

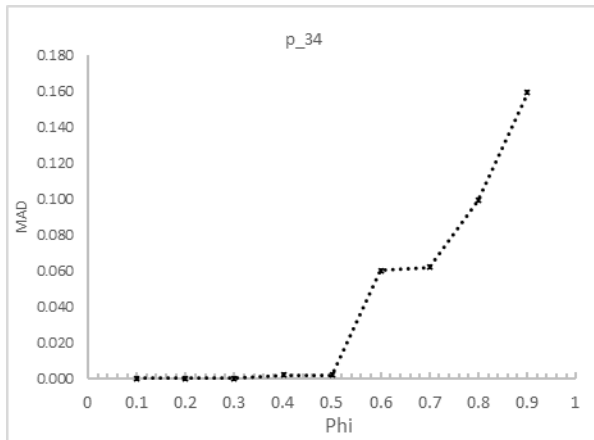


**Figure 2: Mean absolute deviation (MAD) and the measurement errors correlation for path between eta\_2 and eta\_4.**

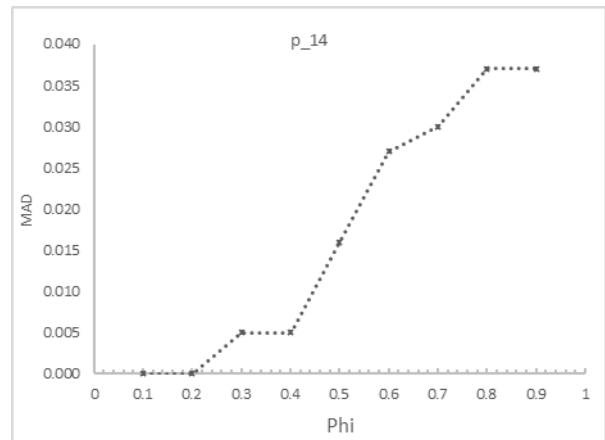
The bias assessment is conducted through the examination of the Measurement Model and Structural Model of the PLSc technique. The Loadings, weights, and Path Coefficients are evaluated using the extent of bias in the estimated parameters. For differing levels of underlying structural parameter values, the MAD values provide insights into the degree of bias present in the parameter estimates. It is discernible that as the magnitude of these parameters increases, there is a corresponding increase in MAD values, indicating an escalation in bias (fig 2). This

biasedness analysis underscores the importance of comprehending the impact of non-orthogonal measurement errors on the parameter estimation process, and the subsequent implications for the reliability and validity of the PLS technique in empirical applications. For clear depiction. The bias analysis is presented in the graphical form for the structural parameters.

The scenario 1 delves into the consequences of measurement errors across the indicators. The outcomes, as presented in tables 1 and 2 and illustrated in graphical analysis (fig 2,3,4), strongly suggests that parameter estimations within both the outer and inner models lack stability under these circumstances, displaying a high degree of sensitivity. Notably, the outer model's parameter estimates lean toward overestimation, signaling that when errors are correlated among indicator variables, the true impact of parameter estimates becomes biased, potentially leading to type-I errors. The structural model parameters biasedness against the MAD is presented in fig 2,3 and 4.



**Figure 3: (MAD) and error correlation between eta\_3 and eta\_4.**



**Figure 4: MAD and error correlation between eta\_1 and eta\_4**

This observation raises concerns about the possibility of mistakenly considering certain indicator variables as pivotal, even though their significance might not hold in the population model. Similar trends are observed with formative indicators. The coefficient estimates of genuine relationships also display a tendency toward overestimation, amplifying the likelihood of type-I errors and indicating the potential misclassification of certain parameters as crucial contributors, despite their limited influence in the population model. This discrepancy arises from the exaggerated loading values of associated indicators. Furthermore, it is evident that as the degree of association escalates, biasedness proportionally increases. Notably, when estimated parameters align within the acceptance range of the loadings, the extent of biasedness experiences a more pronounced surge, particularly as the association value surpasses the range of 0.6 to 0.9.

Given the findings from the initial case and the behaviors of parameter estimations, the interpretation of coefficients and the determination of an indicator's practical relevance to its corresponding construct become intricate, especially when the estimated value gravitates toward the acceptance region. Regarding biasedness, the structural parameters deviate from the true parameter values, displaying a downward bias. This signifies an underestimation of the true effect, particularly when errors are correlated across indicator variables. The evaluation of bias using mean absolute deviation further indicates a proportional increase in bias as the degree of error correlation rises. As a result, the significance of measurement errors within the model, particularly at this level, cannot be overstated. This case underscores the imperative nature of acknowledging measurement errors even within the context of PLS-SEM and it has serious implications for the theory testing.

The scenario 2 of correlation between  $\eta_1$  and  $\eta_2$  is bit different from the other cases. This variance specifically underscores the classical manifestation of multicollinearity arising due to measurement errors. While the underlying effect resembles other instances, the degree of impact differs. Initially, when there's minimal association between the interconnected underlying constructs, the resultant effect is also minimal. However, as this association surpasses 0.6 and advances to 0.8, the consequences become detrimental. This is evidenced by wider confidence intervals and amplified standard error values, which consequently contribute to type-II errors. Consequently, some effects that are significant within the population might not be deemed significant due to these errors.

This trend becomes even more pronounced as the association level climbs to 0.90 or beyond. Consequently, imprecise estimations of coefficients and standard errors emerge as products of this type of measurement inaccuracy. Through the lens of Mean Absolute Deviation (MAD) analysis, the bias computed in relation to the correlation between latent variables and their effects accentuates the importance of cautious handling of correlated latent variables. This suggests that attention is warranted when dealing with non-orthogonal indicators to ensure accurate results. The case of non-orthogonal latent variables is different from the measurement errors correlation.

**Table 3**  
**Results of the Simulation for Scenario 2**

		Scenario2: Non-Orthogonal latent variables due to Measurement errors across Measurement Model																Structural Model Path Coefficients		
		Loadings										weights						$p_{14}$	$p_{24}$	$p_{34}$
$\phi$	PLS c	$\lambda_{11}$	$\lambda_{12}$	$\lambda_{21}$	$\lambda_{22}$	$\lambda_{23}$	$\lambda_{24}$	$\lambda_{25}$	$\lambda_{26}$	$\lambda_{31}$	$\lambda_{32}$	$\lambda_{41}$	$\lambda_{42}$	$\lambda_{43}$	$\lambda_{44}$	$\lambda_{45}$	$\lambda_{46}$			
		0.1		0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.8	0.178
		50	00	50	01	50	20	50	80	00	50	50	00	50	00	50	20		70	10
0.2		0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.8	0.178	0.5	0.7
		52	00	50	01	53	23	50	80	00	50	50	00	50	00	50	20		70	30
0.3		0.4	0.5	0.5	0.5	0.4	0.5	0.5	0.5	0.5	0.4	0.5	0.7	0.8	0.6	0.8	0.8	0.179	0.5	0.7
		76	12	50	88	77	28	48	69	11	48	75	85	50	05	53	22		80	65
0.4		0.4	0.5	0.5	0.3	0.3	0.4	0.5	0.5	0.5	0.4	0.5	0.7	0.8	0.6	0.8	0.8	0.185	0.5	0.7
		87	14	53	55	75	04	51	72	12	48	76	87	50	05	53	22		84	70
0.5		0.4	0.5	0.5	0.3	0.3	0.4	0.5	0.5	0.5	0.4	0.5	0.7	0.8	0.6	0.8	0.8	0.190	0.5	0.7
		88	15	53	56	76	04	79	72	14	50	90	91	51	07	54	25		83	75
0.6		0.5	0.5	0.5	0.4	0.3	0.4	0.5	0.5	0.5	0.4	0.5	0.7	0.8	0.6	0.8	0.8	0.195	0.6	0.8
		48	15	56	00	76	06	84	91	17	50	89	91	51	08	58	29		03	10
0.7		0.5	0.5	0.5	0.4	0.3	0.4	0.5	0.6	0.5	0.5	0.4	0.5	0.7	0.8	0.6	0.8	0.199	0.6	0.9
		53	18	56	04	78	06	99	06	17	51	99	91	54	10	57	29		32	32
0.8		0.5	0.5	0.5	0.4	0.3	0.4	0.6	0.6	0.5	0.5	0.4	0.5	0.7	0.8	0.6	0.8	0.207	0.6	0.9
		54	31	61	04	87	09	11	07	18	55	99	95	55	12	78	30		78	65
0.9		0.5	0.5	0.5	0.4	0.3	0.4	0.6	0.6	0.5	0.5	0.5	0.7	0.8	0.6	0.8	0.8	0.230	0.7	0.9
		54	34	62	04	90	10	15	07	18	56	26	94	54	14	79	30		69	80
Pop. Parameter		0.6	0.7	0.7	0.8	0.6	0.7	0.7	0.7	0.7	0.6	0.8	0.7	0.8	0.6	0.8	0.8	0.17	0.5	0.7
		50	00	50	00	50	20	50	80	00	50	50	00	50	00	50	20		60	0

Sample size N= 400, Number of Simulations R= 50,000 and Phi ( $\phi$ ) is the strength of association between the measurement errors.

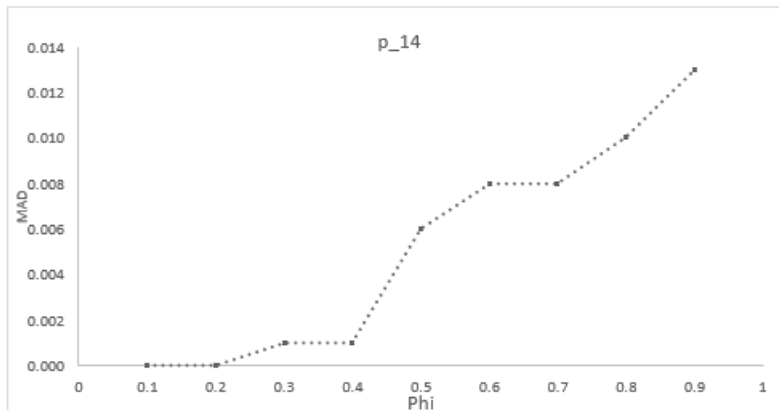
The empirical evidence suggests that in cases where there is high-rate non-orthogonality between indicators, factor loadings behaved erroneously. In some part of the model, they are underestimated while in other are overestimated. This is because the shared variance between indicators can lead to an apparent reduction in the unique variance that each indicator contributes to the latent construct.

As a result, factor loadings might appear lower than they truly are, giving the impression that the indicators are less related to the construct than they are in the population model. Similarly, On the other hand, overestimation of factor loadings occurred when the collinearity between indicators artificially inflates the variance associated with the latent construct. As a result, factor loadings might appear higher than they would be in the case of orthogonal variables, suggesting stronger relationships between indicators and the latent construct than

they are in the case of population parameters, this happens with the  $\lambda_{43}$ ,  $\lambda_{44}$ ,  $\lambda_{45}$  and  $\lambda_{46}$  respectively.

**Table 4**  
**Biasedness Analysis for Scenario 2**

		Measurement Model															Structural Model Path Coefficients			
		Loadings										weights					$p_{14}$	$p_{24}$	$p_{34}$	
$\phi$	PL Sc	$\lambda_{11}$	$\lambda_{12}$	$\lambda_{21}$	$\lambda_{22}$	$\lambda_{23}$	$\lambda_{24}$	$\lambda_{25}$	$\lambda_{26}$	$\lambda_{31}$	$\lambda_{32}$	$\lambda_{41}$	$\lambda_{42}$	$\lambda_{43}$	$\lambda_{44}$	$\lambda_{45}$	$\lambda_{46}$	$p_{14}$	$p_{24}$	$p_{34}$
		0.		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0
1		00	00	00	01	00	00	00	00	00	00	00	00	00	00	00	0	00	00	00
0.		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.0	0.0
2		02	00	00	01	03	03	00	00	00	00	00	00	00	00	0	00	00	00	
0.		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.0	0.0
3		02	01	03	05	04	03	00	00	00	00	01	01	00	05	03	2	05	00	00
0.		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.0	0.0
4		17	04	08	07	03	18	05	03	01	00	02	04	00	05	03	2	05	47	02
0.		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.1	0.0
5		18	06	08	08	05	08	43	03	04	03	21	09	01	07	04	5	16	41	02
0.		0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.1	0.0
6		01	06	11	10	08	11	50	30	08	03	20	10	01	08	08	9	27	48	60
0.		0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.1	0.0
7		08	10	11	13	11	14	71	50	08	05	33	10	04	10	07	9	30	52	62
0.		0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.0	0.1	0.0
8		09	27	18	21	13	16	87	51	10	10	33	15	05	12	28	0	37	55	99
0.		0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.0	0.1	0.1
9		09	31	20	25	17	18	93	51	10	11	71	14	04	14	29	0	37	60	59



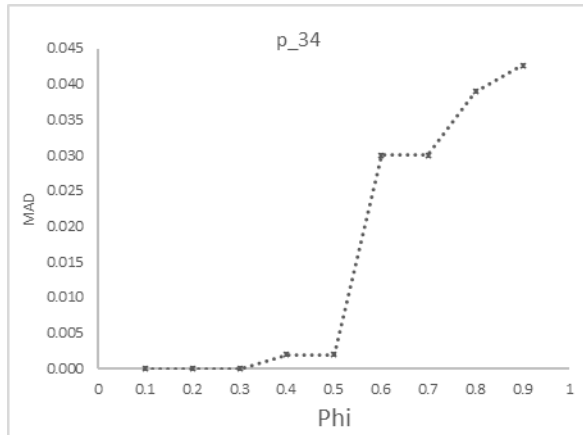
**Figure 5: MAD and strength of non-orthogonality for p\_14**

Overall, this, suggests that the correlated latent variables lead to unstable and unrealistic factor loadings and structural model parameters.

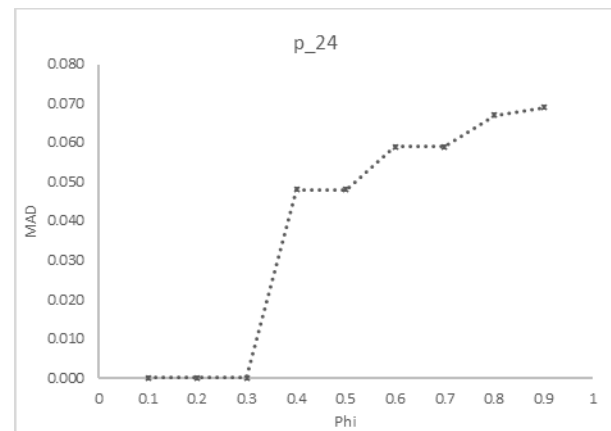
The Mean Absolute Deviation (MAD) values of the estimated parameters provide further insight into this phenomenon. As the degree of association among variables changes, the parameters exhibit heightened sensitivity to variations. This sensitivity is particularly noticeable when the parameter values respond minimally to alterations in association strength. However, in a significant number of cases, the opposite trend prevails, where the extent of bias increases concomitantly with shifts in association. This highlights the remarkable influence of these measurement errors on the parameter estimates. Delving deeper, the MAD values offer a comprehensive perspective on the effect of varying association degrees.

When parameter estimates exhibit a decrease in bias, it generally implies that the inherent nature of these parameters restrains their responsiveness to changes in association. This restraint might be attributed to the robustness of the underlying relationships, or the specific

characteristics of the constructs being measured. On the other hand, most scenarios depict a rather contrasting picture (see figure, 5,6 and 7): a rise in bias proportional to fluctuations in association strength. This phenomenon underscores the delicate nature of these measurement errors and how they can substantially skew the perceived relationships between variables. Such bias amplification is particularly pronounced in cases where the measurement errors are more prevalent or pronounced. In essence, the relationship between parameter estimates and the degree of association is nuanced and multifaceted. This underscores the critical importance of rigorous measurement and careful consideration of the potential impact of measurement errors in the pursuit of accurate and reliable results in various analyses and models.



**Figure 6: trends in MAD against the non-orthogonality strength for p\_24**



**Figure 7: MAD and Phi relationship for p\_34**

## 5. Conclusion

Present study is an attempt to quantify the effects of measurement errors on parameters estimates of the measurement model and structural relationships in the structural model in the framework of PLS-SEM. The result of study unveils the crucial insights into the intricate interplay between the measurement errors, parameter estimates and degree of association among the latent variables. Through an in-depth exploration of the mean absolute deviation values, the comprehensive understanding of the degrees of association on the sensitivity of estimated parameters has been presented. The findings/analysis vividly illustrates that the influence of association strength on parameter estimates is far from uniform. The data reveal a dualistic pattern: parameters exhibit heightened sensitivity to association changes in some instances, while in others, the sensitivity is notably subdued. This phenomenon is indicative of the underlying complexity and multifaceted nature of the relationships between variables.

Furthermore, the phenomenon of bias in parameter estimates emerges as a pivotal factor in this context. We observed a parallel trend between bias and sensitivity, with a notable correlation between the two. Instances where the parameter values are minimally affected by changes in association tend to exhibit decreased bias, reflecting the robustness of these parameters. Conversely, a significant portion of cases demonstrates an exacerbation of bias as the association strength fluctuates. This emphasizes the pivotal role of measurement errors in influencing the parameter estimates and consequently the validity of the overall model.

In essence, our study underscores the importance of meticulously considering the presence of correlated factors, formative indicators, and latent variables when analysing parameter estimates in structural equation modelling. The implications are vast and have far-reaching consequences for diverse fields where these models are applied. As we move

forward, the findings compel researchers to adopt a meticulous approach, accounting for measurement errors and considering the intricate relationships between variables to ensure accurate and reliable results in their analyses.

Historically, researchers have frequently operated under the assumption that Structural Equation Modelling (SEM), due to its ability to account for measurement error and adjust for path attenuation, alleviates concerns related to measurement errors unreliability. However, our study's findings unequivocally demonstrate the invalidity of this assumption in PLS-SEM. Although SEM offers an improvement over techniques like regression that disregard measurement error, resulting in inconsistent parameter estimations, our research underscores the substantial impact of measurement error on the accuracy of estimations for coefficients in both the models. This, consequently, escalates the likelihood of parameters interpretation especially where, the concern is to test the theory.

The implications of these findings are far-reaching. The importance of employing reliable measures remains unparalleled, emphasizing that researchers must conscientiously prioritize the utilization of dependable measurement instruments. This practice, while crucial, can greatly contribute to the enhancement of the integrity and robustness of research outcomes in PLS-SEM.

### **5.1. Way forward**

Given the potential implications of non-orthogonal measurement errors, it is therefore, imperative for the researchers to consider these findings before explicitly drawing results from the PLS-SEM models. The presence of correlated measurement errors carries significant implications, particularly with respect to parameter estimates and inherent bias within the specified conditions. Considering the ongoing advancements and refinements in PLS-SEM methods, it would be prudent to develop an appropriate strategy that addresses the correlations among these errors, much like the strategies used in classical econometrics. Adopting robust techniques such as ridge regression or shrinkage methodologies within the PLS-SEM framework can effectively account for these error correlations

#### **Authors Contribution:**

Rizwan Ahmad: Manuscript preparation, introduction, literature review, methodology and simulations

Ahsan Satti. Thorough review and directions for implementing simulations

#### **Conflict of Interests/Disclosures**

The authors declared no potential conflicts of interest w.r.t the research, authorship and/or publication of this article.

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